American Institute of Certified Public Accountants
National Business Valuation Conference

Cost of Capital

Presented By

Harold G. Martin, Jr., MBA, CPA, ABV, ASA, CFE
Ronald L. Seigneur, MBA, CPA, ABV, CVA

December 4, 2001
I. FUNDAMENTALS

A. General Theory of the Income Approach

1. The value of an investment is equal to the sum of the values of the future benefits discounted to a present value at a rate that reflects the time value of money and degree of risk.¹

2. Investment = equity or invested capital

3. Expected future benefits = “economic income” (the benefit stream that can be converted into value).

4. Discount Rate = yield rate to convert expected future benefits into present value (i.e., cost of capital).

5. Basic formula:⁴

\[
PV = \frac{NCF_n}{(1 + k)^n}
\]

Where:

- \( PV \) = Present value
- \( NCF_n \) = Expected net cash flow
- \( k \) = Discount rate
- \( n \) = nth year

B. Investment in Company

1. The investment in a company is represented by its capital structure which is comprised of the following:

   a) Long-term interest bearing debt
   b) Preferred equity
   c) Common equity

2. Alternative models for valuing investment in company:

   a) Valuation of equity
   b) Valuation of invested capital:

   Market value of invested capital

---


³ The notation used in this presentation is primarily based upon that constructed by Shannon Pratt, Robert Reilly, and Robert Schweihs in *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 3rd Edition, 1996, and adopted for use by the authors of the AICPA business valuation curriculum.
- Interest bearing debt and senior equity
  = Market value of equity

C. Economic Income Streams

1. Economic income – benefit stream that can be converted into value (e.g., net income, EBIT, EBITDA, net cash flow).

2. For purposes of calculating value, generally use net cash flow.
   a) Net cash flow represents cash that is available to pay the owners of capital.
   b) Ibbotson Associates SBBI data are based upon net cash flow to equity.

3. Alternative definitions of net cash flow (based on selected valuation model):
   a) Net cash flow to equity
   b) Net cash flow to invested capital

4. Net cash flow to equity
   a) Those cash flows available to pay out to equity holders (in the form of dividends) after funding operations of the business enterprise, making necessary capital investments, and increasing or decreasing debt financing.\(^4\)
   
   b) Definition:
      Net income
      + Non-cash charges (depreciation, amortization, deferred revenue, and deferred taxes)
      +/- Incremental working capital requirements
      +/- Capital expenditures
      + New long term debt
      - Repayment of long term debt
      - Dividends paid to preferred stock of senior securities
      = Net cash flow to common equity\(^5\)

5. Net cash flow to invested capital
   a) Those cash flows available to pay out to equity holders (in the form of dividends) and debt investors (in the form of principal and interest) after funding operations of the

\(^4\) International Glossary of Business Valuation Terms.
business enterprise and making necessary capital investments.\footnote{International Glossary of Business Valuation Terms.}

b) Definition:
Net income to invested capital (excludes interest expense on long term interest bearing debt)
+ Non-cash charges (depreciation, amortization, deferred revenue, and deferred taxes)
-/+ Incremental working capital requirements
-/+ Capital expenditures
+ Dividends paid to preferred stock of senior securities
= Net cash flow to invested capital\footnote{Adams, Fundamentals of Business Valuation – Part I 5-32.}

D. Discount Rate = Cost of Capital
1. The cost of capital is “the expected rate of return that the market requires in order to attract funds to a particular investment.”\footnote{International Glossary of Business Valuation Terms.}
2. In economic terms, it is an opportunity cost – the cost of foregoing the next best alternative investment.\footnote{Shannon P. Pratt, Cost of Capital – Estimation and Applications (New York, NY: John Wiley & Sons, Inc., 1998) 3.}
3. The cost of capital can be viewed from three different perspectives, each of which represents the same rate:
   a) Company’s assets - discount rate used to present value future cash flows representing
   b) Company’s capital - economic cost of attracting capital (debt and equity)
4. The cost of capital is forward-looking.
5. Cost of capital is based on market value as opposed to historical costs.
6. Cost of capital represents a nominal rate (i.e., includes inflation).
II. THE INCOME APPROACH

A. Single-Stage Model (Capitalization of Economic Income)

1. General theory: if an investment can be expected to produce a stable economic income stream that will grow at a long-term sustainable rate, the value can be calculated by capitalizing the economic income stream.

2. Assumptions
   a) Stable earnings – calculation requires that the economic income stream being capitalized approximate the return for each future year.
   b) Constant growth rate – the economic income is expected to grow at a constant long-term average compounded growth rate.

3. Formula:

\[
PV = \frac{NCF_0 (1 + g)}{(k - g)}
\]

Where:
- PV = Present value
- NCF\(_n\) = Net cash flow (if projected cash flow is used, it is unnecessary to multiply by \((1 + g)\))
- k = Discount rate
- g = Long term average annual compounded growth rate
- n = Year 0

B. Two-Stage Model (Discounted Economic Income)

1. General theory: if an investment cannot be expected to produce a stable economic income stream that will grow at a long-term sustainable rate, the value must be calculated by forecasting the economic income stream for each period until the stream or growth have stabilized.

2. Assumptions
   a) Current economic income or rates of growth are not stable.
   b) At some future point economic income or the rate of growth will stabilize (permitting the use of a capitalization model for calculating the value of the income stream into perpetuity).

---

3. Formula\textsuperscript{12}

\[
PV = \frac{NCF_1}{(1 + k)^1} + \frac{NCF_2}{(1 - k)^2} + \frac{NCF_3}{(1 + k)^3} + \ldots + \frac{NCF_n}{(1 + k)^n} + \frac{NCF_n(1 + g)}{(k - g)^n}\]

\[
\frac{(1 + k)^n}{(1 + k)^n}
\]

Where:

\begin{align*}
PV & = \text{Present value} \\
NCF_n & = \text{Net cash flow for the nth year} \\
k & = \text{Discount rate} \\
g & = \text{Long term average annual compounded growth rate} \\
n & = \text{n\textsuperscript{th} year}\textsuperscript{13}
\end{align*}

C. Other Growth Models

1. Three Stage Growth Model

III. RISK AND THE COST OF CAPITAL

A. Risk

1. “degree of uncertainty as to the realization of expected future economic income”\textsuperscript{14}

B. Types of Risk

1. Maturity risk: “risk that the value of the investment may go up or down because of changes in the general level of interest rates.”\textsuperscript{15}

2. Systematic risk (market risk): “uncertainty of future returns because of the sensitivity of the return on a subject investment to movements in return for the investment market as a whole.”\textsuperscript{16}

3. Unsystematic risk (specific risk): “the uncertainty of expected returns arising from factors other than the market itself.”\textsuperscript{17}

C. Impact of Risk on the Cost of Capital

1. Cost of capital is comprised of two components:

   a) Riskless rate – compensation to investors for renting their money.

\textsuperscript{12} Adams, \textit{Fundamentals of Business Valuation – Part I} 5-20.

\textsuperscript{13} The model as presented uses the “end-of-year” convention which assumes that the economic income is received at the end of each period. An alternative method is the “mid-year” convention which assumes that the economic income is received proportionately during a period.


\textsuperscript{15} Pratt, \textit{Cost of Capital – Estimation and Applications} 36.

\textsuperscript{16} Pratt, \textit{Cost of Capital – Estimation and Applications} 36.

\textsuperscript{17} Pratt, \textit{Cost of Capital – Estimation and Applications} 37.
b) Risk premium – compensation to investors for uncertainty of expected returns.

2. The higher the perception of risk for a particular investment, the higher the rate of return (discount rate) required by the market.

3. The higher the required rate of return, the lower the present value of the investment.\(^{18}\)

IV. MODELS FOR CALCULATING THE COST OF EQUITY

A. Capital Asset Pricing Model

1. Model\(^ {19}\)

\[
E(R_i) = R_f + (RP_m)B + RP_s + RP_u
\]

Where:
- \(E(R_i)\) = Expected rate of return
- \(R_f\) = Risk-free rate of return
- \(RP_m\) = Equity risk premium (market risk)
- \(B\) = Beta
- \(RP_s\) = Size premium
- \(RP_u\) = Specific company risk premium (unsystematic risk)

B. Assumptions Underlying CAPM

1. Investors are risk averse.
2. Investors seek to hold diversified portfolios.
3. Investors have the same expected holding periods.
4. Investors have similar expectations for rates of return.
5. There are no transaction costs.
6. There are no investment-related taxes.
7. The lending rate equals the borrowing rate.
8. Investors can buy or sell any fractional interest.\(^ {20}\)

\(^{19}\) Pratt, Cost of Capital – Estimation and Applications 76.
\(^{20}\) Pratt, Cost of Capital – Estimation and Applications 78.
C. Risk Free Rate of Return
1. Rate investor could receive from an investment free of risk of default.
2. Components: rental rate, inflation, maturity risk (risk that principal’s market value will fluctuate due to changes in interest rates).
3. The common proxy is yield to maturity on U.S. Treasury Bond with maturity of 20 years as of the valuation date. This maturity is used because the Ibbotson data used to derive the equity risk premium begins in 1926 and 20 years was the longest maturity on a U.S. Treasury security at that time.21

D. Equity Risk Premium
1. Additional risk associated with investing in a portfolio of large publicly traded common stocks over the risk free rate of return.
2. Ibbotson calculates as follows for 2001:22
   
   \[
   \text{Market Total Return for large stock index}^* \times 0.1298 \\
   \text{Risk-Free Rate} \times (0.0522) \\
   \text{Long-Horizon Equity Risk Premium} \times 0.0776
   \]
   
   * Represented by S&P 500 which is considered to represent the market as a whole.
3. Return on equity is comprised of distributions during the holding period and capital gain and losses.

E. Beta
1. Measure of systematic risk: “uncertainty of future returns owing to the sensitivity of the return on the subject investment to movements in the returns for a composite measure of marketable securities.”23
2. Beta is calculated by comparing the excess return for a security to the excess return for a market index. Excess returns are defined as total returns (including dividends and capital gains/losses) over the returns from a risk-free investment.
3. Interpretation of Beta Values

---

21 Pratt, Cost of Capital – Estimation and Applications 60.
> 1.0 If market rate of return increases or decreases, the stock rate of return moves in same direction with greater magnitude.

= 1.0 If market rate of return increases or decreases, the stock rate of return moves in same direction with equal magnitude.

< 1.0 If market rate of return increases or decreases, the stock rate of return moves in same direction with lower magnitude.

Negative If market rate of return increases or decreases, the stock rate of return moves in opposite direction.24

4. Sources:
   a) Selected guideline public companies
   b) Ibbotson Associates Beta Book
   c) Ibbotson Associates Cost of Capital Quarterly.

F. Size Premium

1. Additional risk relating to investing in the common stock of smaller public companies.

2. Sources:
   a) Ibbotson Associates SBBI Yearbook
   b) PricewaterhouseCoopers studies

G. Specific Company Risk (Unsystematic Risk)

1. Additional risk relating to the subject company’s characteristics that are not reflected in the equity risk premium or size premium.

2. Representative factors to consider:
   a) Depth of management
   b) Importance of key personnel
   c) Stability of industry
   d) Diversification of customer base
   e) Diversification of product line
   f) Diversification / stability of suppliers
   g) Geographic location
   h) Stability of earnings
   i) Earnings margins

j) Financial structure

3. Source: Appraiser’s professional judgment.

H. Build-Up Model

1. Traditional model

\[ E(R_i) = R_f + R_{Pm} + R_{Ps} + R_{Pu} \]

Where:
- \( E(R_i) \) = Expected rate of return
- \( R_f \) = Risk-free rate of return
- \( R_{Pm} \) = Equity risk premium (market risk)
- \( R_{Ps} \) = Size premium
- \( R_{Pu} \) = Specific company risk premium (unsystematic risk)

a) Build-Up Model eliminates Beta.

b) \( R_{Pu} \) in the Build-Up Model includes the additional risk captured in Beta in the CAPM model.

2. New model using Ibbotson industry risk premia

\[ E(R_i) = R_f + R_{Pm} + R_{Ps} +/- R_{Pi} + R_{Pu} \]

Where:
- \( E(R_i) \) = Expected rate of return
- \( R_f \) = Risk-free rate of return
- \( R_{Pm} \) = Equity risk premium (market risk)
- \( R_{Ps} \) = Size premium
- \( R_{Pi} \) = Industry risk premium
- \( R_{Pu} \) = Specific company risk premium (unsystematic risk)

a) The *Ibbotson SBBI Yearbook Valuation Edition* now provides a table of industry risk premia to help eliminate some of the subjectivity in calculating the \( R_{Pu} \). An excerpt of industry premia for the services industry is presented in the following table:

---


Table 2-5
Excerpt of Industry Premia Estimates

<table>
<thead>
<tr>
<th>SIC Code</th>
<th>Services</th>
<th>Number of Companies</th>
<th>Industry Premia</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>Hotels, Rooming Houses, and Other Lodging Places</td>
<td>39</td>
<td>-2.43%</td>
</tr>
<tr>
<td>701</td>
<td>Hotels and Motels</td>
<td>37</td>
<td>-2.41</td>
</tr>
<tr>
<td>72</td>
<td>Personal Services</td>
<td>19</td>
<td>4.96</td>
</tr>
<tr>
<td>721</td>
<td>Laundry, Cleaning, and Garment Services</td>
<td>5</td>
<td>-3.51</td>
</tr>
<tr>
<td>729</td>
<td>Miscellaneous Personal Services</td>
<td>7</td>
<td>8.75</td>
</tr>
<tr>
<td>73</td>
<td>Business Services</td>
<td>812</td>
<td>4.60</td>
</tr>
<tr>
<td>731</td>
<td>Advertising</td>
<td>22</td>
<td>0.42</td>
</tr>
<tr>
<td>732</td>
<td>Consumer Credit Reporting Agencies, Mercantile Reporting Agencies, and</td>
<td>6</td>
<td>-4.53</td>
</tr>
<tr>
<td>733</td>
<td>Mailing, Reproduction, Commercial Art, and Stenographic Services</td>
<td>14</td>
<td>1.93</td>
</tr>
<tr>
<td>735</td>
<td>Miscellaneous Equipment Rental and leasing</td>
<td>40</td>
<td>2.14</td>
</tr>
<tr>
<td>736</td>
<td>Personnel Supply Services</td>
<td>50</td>
<td>-3.58</td>
</tr>
<tr>
<td>737</td>
<td>Computer Programming, Data Processing, and Other Computer Services</td>
<td>605</td>
<td>4.99</td>
</tr>
</tbody>
</table>

V. MODEL FOR CALCULATING THE COST OF INVESTED CAPITAL

A. Weighted Average Cost of Capital (WACC)

1. The WACC is a blend of the subject company’s cost of debt and cost of equity weighted by the respective market values of debt and equity.

2. The cost of equity for a closely held company is calculated using either the CAPM or Build-Up Models.

3. The cost of debt is based on the subject company’s actual cost.

4. Model

\[
WACC = (k_e \times W_e) + (k_p \times W_p) + \left( k_{dp(t)} \times (1 - t) \times W_d \right)
\]

Where:

- \( WACC \) = Weighted average cost of capital
- \( k_e \) = Cost of common equity
- \( W_e \) = Weight of common equity at market
- \( k_p \) = Cost of preferred equity


38.

\[ W_p = \text{Weight of preferred equity at market} \]
\[ k_{d(pt)} = \text{Pre-tax cost of debt} \]
\[ t = \text{Subject company tax rate} \]
\[ W_d = \text{Weight of long term interest bearing debt at market} \]

B. Iterative Method for Deriving FMV of Equity for Computing the WACC

1. See Appendix A.

VI. ANOTHER LOOK AT SELECTED ISSUES

A. Economic Income Stream to be Discounted

1. Ibbotson notes that the appropriate cash flow to be valued is “free cash flow” or cash flow to invested capital.\(^{30}\)

2. This definition of cash flow is appropriate because:
   a) Represents cash flows to total invested capital allowing the owners to determine how to allocate the cash flows (i.e., to debt or equity).
   b) Ibbotson data used to develop the cost of equity is applicable to net cash flow.

3. Net income is the incorrect economic income stream because it includes certain accounting adjustments and includes the impact of the company’s capital structure.\(^{31}\) (It should be noted that there may occasions where net income approximates net cash flow because of the characteristics of the subject company).

4. The “expected cash flow” calculated using probability-weighted projections of cash flows should be used for valuation purposes instead of the “most likely” cash flows.\(^{32} \)\(^{33}\)

\(^{29}\) I wish to acknowledge the contributions of Michael Mattson of the Financial Valuation Group in Chicago, Illinois, and Mark Zyla of the Philips-Hitchner Group in Atlanta, Georgia, for their suggestions and critique of this model. Any errors relating to its application are solely my own.


B. The Tax Effect

1. The economic income stream and the discount rate must be consistent with respect to taxes.\(^{34}\)

2. Ibbotson data is based on after tax cash flows and therefore a discount rate derived using the Ibbotson data is an after tax rate. Therefore, if pre-tax cash flows are used, then the discount rate must be adjusted to a pre-tax rate. However, there is no simple methodology for accurately adjusting the Ibbotson data to a pretax basis.\(^{35}\)

3. *Gross v. Commissioner* – The matching of the definition of cash flows with the discount rate was a key consideration in this tax court case. The taxpayer’s expert tax affected S Corporations economic income using a C Corporation rate and applied an after-tax discount rate. The IRS’s expert calculated a pre-tax discount rate and applied it to the S Corporation’s pre-tax cash flows. The Court ruled in favor of the IRS. The case is on appeal.

4. Applicable tax rate
   a) The combined federal and state tax rate for the subject company should be used when tax-affecting cash flows.
   b) There are alternatives to calculating the applicable tax rate\(^{36}\)
      
      (3) The marginal statutory tax rate for the subject company (probably used most often in practice)
      
      (4) The expected tax rate for the subject company – Ibbotson, quoting research by Graham, notes that a majority of firms can expect to pay less than the marginal rate. However, at issue is whether or not the subject company being valued can duplicate this.

C. Risk Free Rate of Return

1. As previously noted, the common proxy is yield to maturity on U.S. Treasury Bond with maturity of 20 years as of the valuation date. This maturity is used because the Ibbotson data used to derive the equity risk premium begins in 1926 and 20 years was the longest maturity on a U.S. Treasury security at that time.\(^{37}\)

---


\(^{35}\) Pratt, *Cost of Capital – Estimation and Applications* 112.


\(^{37}\) Pratt, *Cost of Capital – Estimation and Applications* 60.
2. However, on October 31, 2001, the U.S. Treasury Department announced it was suspending the 30-year Treasury bond and instead shifting to the 10 year note as the benchmark bond.  

3. While this announcement should have no immediate impact on the selection of the 20-year yield to maturity as the proxy for the risk free rate of return, it will at some future date.

D. Equity Risk Premium

1. Arithmetic versus Geometric Mean

   a) Ibbotson’ ERP data are arithmetic average risk premia as opposed to geometric average risk premia.  

   b) Ibbotson believes the arithmetic average is the most appropriate average for use in the CAPM and Build-Up Models to derive the cost of equity applicable to expected future cash flows.

   c) This view is not shared by all practitioners.

   d) Example.

2. Appropriate Historical Time Period for Estimating ERP

   a) Ibbotson ERP covers the time period from 1926 to the present. This is based on the fact that Ibbotson uses the data compiled by the Center for Research in Security Prices which uses data from 1926 because 1) it is the time at which quality financial data became available, and 2) it includes the volatile financial markets of the 1920’s and 1930’s.

   b) Some analysts estimated the ERP using a more recent time horizon on the theory that this period is more indicative of the near future. Further, they believe that the period during 1920 to 1940 contained unusual events that effected the market. However, this ignores the fact that many unusual events have taken place since these earlier period, some of which have resulted in volatile changes in the market.

   c) The following table from Ibbotson illustrates that the longer the time period, the more stable the ERP.

---

41 Pratt, Cost of Capital – Estimation and Applications 63.
Table 4-5  
Stock Market Return and Equity Risk Premium Over Time  
1926-2000

<table>
<thead>
<tr>
<th>Period Length</th>
<th>Period Dates</th>
<th>Large Company Stock Arithmetic Mean Total Return</th>
<th>Long-Horizon Equity Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 years</td>
<td>1926-2000</td>
<td>13.0%</td>
<td>7.8%</td>
</tr>
<tr>
<td>70</td>
<td>1931-2000</td>
<td>13.1</td>
<td>7.7</td>
</tr>
<tr>
<td>60</td>
<td>1941-2000</td>
<td>14.1</td>
<td>8.3</td>
</tr>
<tr>
<td>50</td>
<td>1951-2000</td>
<td>14.5</td>
<td>8.7</td>
</tr>
<tr>
<td>40</td>
<td>1961-2000</td>
<td>13.1</td>
<td>5.8</td>
</tr>
<tr>
<td>30</td>
<td>1971-2000</td>
<td>14.5</td>
<td>6.3</td>
</tr>
<tr>
<td>20</td>
<td>1981-2000</td>
<td>16.5</td>
<td>8.1</td>
</tr>
<tr>
<td>15</td>
<td>1986-2000</td>
<td>16.8</td>
<td>9.5</td>
</tr>
<tr>
<td>10</td>
<td>1991-2000</td>
<td>18.4</td>
<td>11.6</td>
</tr>
<tr>
<td>5</td>
<td>1996-2000</td>
<td>19.4</td>
<td>13.2</td>
</tr>
</tbody>
</table>

E. Beta

1. Sources of beta
   a) The betas for the same stock may differ from one reporting source to another. Therefore, the same source should be used when identifying betas for selected guideline companies.
   b) Ibbotson uses the S&P 500 in its calculation of Beta for the Cost of Capital Yearbook and the Beta Book.

2. Levered and Un-Levered Betas
   a) Published betas for publicly traded equities reflect the actual capital structure of the specific company. These betas are “levered betas” as they reflect the leverage in the company’s capital structure.
   b) If the leverage of the subject company being valued is materially different from the selected guideline publicly traded companies (or other sources such as the Ibbotson Cost of Capital Yearbook or Ibbotson Beta Book), then the beta should be adjusted to eliminate the effect of the differing degrees of leverage.

---

c) The adjustment to remove the effect of the guideline public companies’ leverage from beta and restating the beta to reflect the subject company’s leverage is referred to as “unlevering” and “relevering” the beta.

d) Formula for unlevering beta for guideline public company

\[ B_u = \frac{B_l}{1 + (1 - t) \frac{W_d}{W_e}} \]

Where:
- \( B_u \) = Beta unlevered for guideline public company
- \( B_l \) = Beta levered for guideline public company
- \( t \) = Tax rate for guideline public company
- \( W_e \) = Weight of guideline public company common equity at market
- \( W_d \) = Weight of guideline public company long term interest bearing debt at market

e) Formula for relevering beta for subject company

\[ B_l = B_u \left[ 1 + (1 - t) \frac{W_d}{W_e} \right] \]

Where:
- \( B_l \) = Beta re-levered for subject company
- \( B_u \) = Beta unlevered for guideline public company
- \( t \) = Tax rate for subject company
- \( W_e \) = Weight of subject company common equity at market
- \( W_d \) = Weight of subject company long term interest bearing debt at market

3. Sources

F. Size Premia

1. Ibbotson Studies

   a) Size premia decile groupings

   (1) Ibbotson reports the size premia in ten groupings (or deciles) based on market capitalization. For

---

\[^44\] Pratt, *Cost of Capital – Estimation and Applications* 84.
example, the market capitalization for the largest company in the tenth decile is $84,521,000.45.

(2) Ibbotson also combines selected deciles to into larger groupings: Mid-Cap: Deciles 3-5, Low-Cap Deciles 6-8, Micro-Cap Deciles 9-10

(3) The following table presents the size premium for the 10th decile as well as the Mid, Low, and Micro-Cap groupings.

Table 6-5
Long-Term Returns in Excess of CAPM Estimation for Decile Portfolios of the NYSE/AMEX/NASDAQ
1926-2000

<table>
<thead>
<tr>
<th>Decile</th>
<th>Beta</th>
<th>Actual Arithmetic Mean Return</th>
<th>Actual Realized Return in Excess of Riskless Rate (*)</th>
<th>Estimated Return in Excess of Riskless Rate Using CAPM (**)</th>
<th>Size Premium (Return in Excess of CAPM) (***)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-Smallest</td>
<td>1.42</td>
<td>20.90</td>
<td>15.67</td>
<td>11.05</td>
<td>4.63</td>
</tr>
<tr>
<td>Mid-Cap, 3-5</td>
<td>1.12</td>
<td>14.46</td>
<td>9.23</td>
<td>8.65</td>
<td>0.58</td>
</tr>
<tr>
<td>Low-Cap, 6-8</td>
<td>1.22</td>
<td>15.75</td>
<td>10.52</td>
<td>9.45</td>
<td>1.07</td>
</tr>
<tr>
<td>Micro-Cap, 9-10</td>
<td>1.36</td>
<td>18.41</td>
<td>13.18</td>
<td>10.56</td>
<td>2.62</td>
</tr>
</tbody>
</table>

Calculations:

*Arithmetic mean return (C) 20.90
  Arithmetic mean on 20-year government bond - 5.22
  Actual return in excess of riskless rate (D) 15.68

**Arithmetic mean total return of the S&P 12.98
  Arithmetic mean on 20-year government bond - 5.22
  Equity risk premium 7.76
  Beta x 1.42
  Estimated return in excess of riskless rate using CAPM (E) 11.02

***Actual return in excess of riskless rate (D) 15.68
  Estimated return in excess of riskless rate using CAPM (E) 11.02
  Size premium (return in excess of CAPM) (F) 4.66

b) The CAPM Model and size premia

(1) There is much debate on whether the Micro-Cap or 10th decile should be used for valuing closely held businesses with market capitalizations less than that of the largest company in the 10th decile.

(2) The Micro-Cap includes a broader sample of companies; however, it also includes much larger companies.

(3) The 10th decile, while perhaps presenting a sample of companies that are closer in size to the subject company, still includes many larger companies.

(4) In an effort to address this issue, the *Ibbotson SBBI 2001 Yearbook Valuation Edition* includes a further breakout of the 10th decile into two additional groupings. The market capitalization for the largest company in the 10a grouping is $84,521,000 and for 10b it is $48,345,000. However, the 10b subgrouping is not without some inherent limitations:

(a) Infrequent trading biases

(b) Delisting bias

(c) Impact of trading transaction costs on lower priced stocks in relation to the value of the underlying shares

(5) The following table presents the size premium for the 10a and 10b subgroupings.

---


c) Build-Up Model and size premia

(1) Some practitioners use the same size premia as that used for the CAPM model

(2) However, there is an alternative school that argues that this measure of the size premium is incomplete because the size premium captured in beta for the CAPM Model is not reflected in the Build-Up Model. In order to capture this additional increment in the size premium, it is suggested that the difference between the actual return in excess of the riskless rate and the equity risk premium be used.\textsuperscript{50}

(3) The alternative size premium (non-beta adjusted) would be calculated as follows:

\[
\text{Actual return in excess of riskless rate} \quad 13.18 \\
\text{(Table 6-5, Column C, Micro-cap)} \\
\text{Equity risk premium (Table C-1)} \quad 7.8 \\
\text{Size premium (non-beta adjusted)} \quad 5.38
\]

(4) A problem in using the non-beta adjusted size premium is that this assumes that the subject company has the same systematic risk (beta) as the

---


\textsuperscript{50} Pratt, \textit{Cost of Capital – Estimation and Applications} 110.
2. PricewaterhouseCoopers Studies

a) Roger Grabowski and David King of PWC have expanded the study of the effect of size on the cost of capital and further broken down the size groupings into 25 groups.

b) The PWC study also introduces other criteria in addition to market capitalization:
   (1) Market value of equity
   (2) Book value of equity
   (3) Five-year average net income
   (4) Market value of invested capital
   (5) Book value of invested capital
   (6) Five year average EBITDA
   (7) Sales
   (8) Number of employees

c) Summaries of the PWC size premium studies are presented in Business Valuation Review.52

d) These studies corroborate the Ibbotson data that there is a size effect.

G. WACC – Capital Structure

1. Weighting of Debt and Equity

a) Control interest

   (1) Standard of value = Fair Market Value
      (a) Use subject company’s capital structure.
      (b) Use optimal capital structure as control buyer has ability to change capital structure. (Note: must determine if subject company has ability to attain optimal structure).

   (2) Standard of value = Investment Value
      (a) Use specific owner’s or buyer’s capital structure.

b) Minority interest

   (3) Use subject company’s capital structure as minority interest lacks control to change capital structure.53

---

53 Pratt, Cost of Capital – Estimation and Applications 52.
H. Does the Discount Rate Determine The Type of Ownership Interest?

1. Some analysts believe that because certain data used to calculate the cost of capital is derived from public company data which represents minority interests, the cost of capital reflects a minority interest value.

2. However, the majority of practitioners believe that it is the nature of the cash flows that determine whether or not a control or minority interest value is derived. As Roger Ibbotson has stated:

   When you are purchasing a company you are acquiring the ability to potentially control future cash flows. To acquire this option to exercise control, you must pay a premium. Holding all else constant, it should not impact the discount rate.  

VII. COMMON ERRORS IN CALCULATING THE COST OF CAPITAL

VIII. APPLYING IBBOTSON DATA

A. SBBI Valuation Edition

B. Cost of Capital Yearbook
   1. Business Development Series

C. Beta Book

D. Cost of Capital Web Site

IX. ADDITIONAL READINGS


Mard, Michael J. and James S. Rigby. “New Research to Estimate Cost of Capital.” 
*CPA Expert.* New York, NY: American Institute of Certified Public 

Martin, Jr., Harold G., ed. “Calculating the Size Premium Component of the Cost of 
Equity.” *ABVE-Valuation Alert.* New York, NY: American Institute of 

American Institute of Certified Public Accountants, Summer 1999.

*CPA Expert.* New York, NY: American Institute of Certified Public 


Rigby, James S. and Michael J. Mattson. “Capitalization and Discount Rates: 
Mathematically Related, But Conceptually Different.” *CPA Expert.* New York, 
NY: American Institute of Certified Public Accountants, Fall 1996.

Seigneur, Ronald L. “Rate Determination in Business Valuation Assignments,” qtd. in 
James R. Hitchner, ed. *Financial Valuation Handbook – Consensus View on 
Applications in Business Valuation “Best Practices.”* New York, NY: John 
Wiley & Sons, Inc., publication pending.
Appendix A
Iterative Method for Deriving FMV of Equity
When Computing the WACC in an Invested Capital Model
## Iterative Method for Deriving FMV of Equity for Computing the WACC

### Key Assumptions

<table>
<thead>
<tr>
<th>Subject Company</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation date</td>
<td>December 31, 2000</td>
</tr>
<tr>
<td>Interest to be valued</td>
<td>100% non-marketable equity interest</td>
</tr>
<tr>
<td>Standard of value</td>
<td>Fair market value (excluding discounts/premiums to simplify example)</td>
</tr>
<tr>
<td>Premise of value</td>
<td>Going concern</td>
</tr>
<tr>
<td>Book Value of Debt</td>
<td>$2,000,000</td>
</tr>
<tr>
<td>Book Value of Equity</td>
<td>$4,000,000</td>
</tr>
<tr>
<td>Tax rate (federal and state)</td>
<td>0.40</td>
</tr>
<tr>
<td>Interest rate for long-term interest bearing debt</td>
<td>0.08</td>
</tr>
</tbody>
</table>

### Cost of Capital Variables

| Risk-free rate (1) | 0.0628 |
| Equity risk premium (2) | 0.0810 |
| Beta - unlevered (3) | 1.1200 |
| Risk premium for size (4) | 0.0463 |
| Specific (unsystematic) risk (5) | 0.0200 |

### Other Key Assumptions

- Estimated long term growth rate for net cash flow to invested capital: 0.0700
- Net cash flow to invested capital will stabilize after five years
- Assume mid-year discounting convention.
- All percentages expressed as decimal equivalents.

### Notes

1. Source: Federal Reserve Statistical Release H.15(519) Selected Interest Rates 20 year U.S. Treasury yield as of December 29, 2000 (last day market was open before year end).
5. Judgmentally determined by appraiser.

### Formulas

Discount rate for net cash flow to equity calculated using CAPM as follows:

\[
E(R_i) = R_f + B(R_{Pm}) + R_Ps + R_{Pu}
\]

where:

- \(E(R_i)\) = Expected return on an individual security
- \(R_f\) = Rate of return available on a risk-free security as of the valuation date
- \(B\) = Beta
- \(R_{Pm}\) = Risk premium for equities
- \(B(R_{Pm})\) equals systematic risk
- \(R_Ps\) = Risk premium for size
- \(R_{Pu}\) = Risk premium for specific company (unsystematic risk)

Re-levered beta for subject company

\[
B_L = B_u (1 + (1 - t)W_d / W_e)
\]

where:

- \(B_L\) = Beta re-levered for the subject company
- \(B_u\) = Beta unlevered for industry (or guideline companies)
- \(t\) = tax rate for the subject company
- \(W_d\) = Percent debt in the capital structure for subject company
- \(W_e\) = Percent equity in the capital structure for subject company
- estimated using iterative approach (or target capital structure)
1 Estimation of Cost of Equity Using CAPM

<table>
<thead>
<tr>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate (Rf)</td>
</tr>
<tr>
<td>Systematic risk</td>
</tr>
<tr>
<td>Equity risk premium (RPm) x Beta (B)</td>
</tr>
<tr>
<td>Average comparative company return</td>
</tr>
<tr>
<td>Risk premium for size (RPs)</td>
</tr>
<tr>
<td>Specific (unsystematic) risk (RPu)</td>
</tr>
<tr>
<td>Discount rate for net cash flow (Ke)</td>
</tr>
</tbody>
</table>

2 Calculation of Re-Levered Beta for Subject Company

<table>
<thead>
<tr>
<th>Guideline Co. or Industry Beta Un-Levered</th>
<th>Market Value</th>
<th>Debt</th>
<th>Equity</th>
<th>Total Capital</th>
<th>Tax Rate (t)</th>
<th>Percent of Capital Wd</th>
<th>Percent of Capital We</th>
<th>Subject Company Beta Levered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>2,000,000</td>
<td>4,000,000</td>
<td>6,000,000</td>
<td>0.40</td>
<td>0.3333</td>
<td>0.6667</td>
<td>0.6667</td>
<td>1.46</td>
</tr>
</tbody>
</table>

3 Estimation of Discount and Capitalization Rates for Invested Capital

<table>
<thead>
<tr>
<th>Estimated Market Value</th>
<th>Percent of Capital (W)</th>
<th>Cost of Capital Rate (k)</th>
<th>Tax Rate (t)</th>
<th>Tax-Effect Rate (k(1-t))</th>
<th>Weighted Average Cost WACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation of WACC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt</td>
<td>2,000,000</td>
<td>0.3333</td>
<td>0.08</td>
<td>0.40</td>
<td>0.0480</td>
</tr>
<tr>
<td>Equity</td>
<td>4,000,000</td>
<td>0.6667</td>
<td>0.2474</td>
<td>N/A</td>
<td>0.2474</td>
</tr>
<tr>
<td>Total</td>
<td>6,000,000</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation of Capitalization rate</th>
<th>Discount rate for net cash flow</th>
<th>Less sustainable average growth rate</th>
<th>Capitalization rate for net cash flow for future year</th>
<th>One plus growth rate</th>
<th>Capitalization rate for net cash flow for current year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1809</td>
<td>0.0700</td>
<td>0.1109</td>
<td>1.0700</td>
<td>0.1036</td>
</tr>
</tbody>
</table>

4 Discounted Economic Income Method (Invested Capital Model)

<table>
<thead>
<tr>
<th>12/31/01</th>
<th>12/31/02</th>
<th>12/31/03</th>
<th>12/31/04</th>
<th>12/31/05</th>
<th>Terminal Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net cash flow to invested capital</td>
<td>$500,000</td>
<td>$550,000</td>
<td>$605,000</td>
<td>$665,500</td>
<td>$732,050</td>
</tr>
<tr>
<td>Capitalization rate for net cash flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$7,063,066</td>
</tr>
<tr>
<td>Terminal value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period</td>
<td>0.50</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>4.50</td>
</tr>
<tr>
<td>Present value of cash flows</td>
<td>460,112</td>
<td>428,591</td>
<td>399,229</td>
<td>371,879</td>
<td>346,403</td>
</tr>
<tr>
<td>Market value of invested capital</td>
<td>5,348,428</td>
<td>5,348,428</td>
<td>5,348,428</td>
<td>5,348,428</td>
<td>5,348,428</td>
</tr>
<tr>
<td>Less interest bearing debt</td>
<td>2,000,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of equity</td>
<td>3,348,428</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: As the value of the calculated equity in Step 4 is not equal to the value of the equity estimated to determine the market weight in Step 3, the value of equity in Steps 2 and 3 will be re-set to the equity calculated in Step 4.
### 1 Estimation of Cost of Equity Using CAPM

<table>
<thead>
<tr>
<th>Rate</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate (Rf)</td>
<td>0.0628</td>
</tr>
<tr>
<td>Systematic risk</td>
<td></td>
</tr>
<tr>
<td>Equity risk premium (RPm)</td>
<td>0.0810</td>
</tr>
<tr>
<td>x Beta (B)</td>
<td>1.52</td>
</tr>
<tr>
<td>Average comparative company return</td>
<td>0.1231</td>
</tr>
<tr>
<td>Risk premium for size (RPs)</td>
<td>0.0463</td>
</tr>
<tr>
<td>Specific (unsystematic) risk (RPu)</td>
<td>0.0200</td>
</tr>
<tr>
<td>Discount rate for net cash flow (Ke)</td>
<td>0.2522</td>
</tr>
</tbody>
</table>

### 2 Calculation of Re-Levered Beta for Subject Company

<table>
<thead>
<tr>
<th>Guideline Co. or Industry Beta Un-Levered</th>
<th>Market Value</th>
<th>Debt</th>
<th>Equity</th>
<th>Total Capital</th>
<th>Tax Rate (t)</th>
<th>Percent of Capital Wd</th>
<th>Percent of Capital We</th>
<th>Subject Company Beta Levered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guideline Co. or Industry Beta Un-Levered</td>
<td>1.12</td>
<td>2,000,000</td>
<td>3,348,000</td>
<td>5,348,000</td>
<td>0.40</td>
<td>0.3740</td>
<td>0.6260</td>
<td>1.52</td>
</tr>
</tbody>
</table>

### 3 Estimation of Discount and Capitalization Rates for Invested Capital

<table>
<thead>
<tr>
<th>Estimated Market Value</th>
<th>Percent of Capital (W)</th>
<th>Cost of Capital Rate (k)</th>
<th>Tax Rate (t)</th>
<th>Tax-Effective Rate k[1-t]</th>
<th>Weighted Average Cost WACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>2,000,000</td>
<td>0.3740</td>
<td>0.08</td>
<td>0.40</td>
<td>0.0480</td>
</tr>
<tr>
<td>Equity</td>
<td>3,348,000</td>
<td>0.6260</td>
<td>0.2522</td>
<td>N/A</td>
<td>0.2522</td>
</tr>
<tr>
<td>Total</td>
<td>5,348,000</td>
<td>1.0000</td>
<td></td>
<td></td>
<td>0.1759</td>
</tr>
</tbody>
</table>

#### Calculation of WACC
- Discount rate for net cash flow: 0.1759
- Less sustainable average growth rate: 0.0700
- Capitalization rate for net cash flow for future year: 0.1059
- One plus growth rate: 1.0700
- Capitalization rate for net cash flow for current year: 0.0990

#### Calculation of Capitalization rate
- Discount rate for net cash flow: 0.1759
- Less sustainable average growth rate: 0.0700
- Capitalization rate for net cash flow for future year: 0.1059
- One plus growth rate: 1.0700
- Capitalization rate for net cash flow for current year: 0.0990

### 4 Discounted Economic Income Method (Invested Capital Model)

<table>
<thead>
<tr>
<th>Terminal Period</th>
<th>12/31/01</th>
<th>12/31/02</th>
<th>12/31/03</th>
<th>12/31/04</th>
<th>12/31/05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net cash flow to invested capital</td>
<td>$500,000</td>
<td>$550,000</td>
<td>$605,000</td>
<td>$665,500</td>
<td>$732,050</td>
</tr>
<tr>
<td>Capitalization rate for net cash flow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$783,294</td>
</tr>
<tr>
<td>Terminal value Period</td>
<td>0.50</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>4.50</td>
</tr>
<tr>
<td>Present value of cash flows</td>
<td>461,089</td>
<td>431,327</td>
<td>403,487</td>
<td>377,443</td>
<td>353,081</td>
</tr>
<tr>
<td>Market value of invested capital</td>
<td>5,593,911</td>
<td>431,327</td>
<td>403,487</td>
<td>377,443</td>
<td>353,081</td>
</tr>
<tr>
<td>Less interest bearing debt</td>
<td>2,000,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of equity</td>
<td>3,593,911</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: As the value of the calculated equity in Step 4 is not equal to the value of the equity estimated to determine the market weight in Step 3, the value of equity in Steps 2 and 3 will be re-set to the equity calculated in Step 4.
1 Estimation of Cost of Equity Using CAPM

<table>
<thead>
<tr>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-free rate (Rf)</td>
</tr>
<tr>
<td>Systematic risk</td>
</tr>
<tr>
<td>Equity risk premium (RPm)</td>
</tr>
<tr>
<td>x Beta (B)</td>
</tr>
<tr>
<td>Average comparative company return</td>
</tr>
<tr>
<td>Risk premium for size (RPs)</td>
</tr>
<tr>
<td>Specific (unsystematic) risk (RPu)</td>
</tr>
<tr>
<td>Discount rate for net cash flow (Ke)</td>
</tr>
</tbody>
</table>

2 Calculation of Re-Levered Beta for Subject Company

<table>
<thead>
<tr>
<th>Guideline Co. or Industry Beta Un-Levered</th>
<th>Market Value</th>
<th>Debt</th>
<th>Equity</th>
<th>Total Capital</th>
<th>Tax Rate (t)</th>
<th>Percent of Capital Wd</th>
<th>Percent of Capital We</th>
<th>Leverage Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.12</td>
<td>2,000,000</td>
<td></td>
<td></td>
<td>3,594,000</td>
<td>0.40</td>
<td>0.3575</td>
<td>0.6425</td>
<td>1.49</td>
</tr>
</tbody>
</table>

3 Estimation of Discount and Capitalization Rates for Invested Capital

<table>
<thead>
<tr>
<th>Calculated of WACC</th>
<th>Estimated Market Value</th>
<th>Percent of Capital (W)</th>
<th>Cost of Capital Rate (k)</th>
<th>Tax Rate (t)</th>
<th>Tax-Effect Rate k[1-t]</th>
<th>Weighted Average Cost WACC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>2,000,000</td>
<td>0.3575</td>
<td>0.08</td>
<td>0.40</td>
<td>0.0480</td>
<td>0.0172</td>
</tr>
<tr>
<td>Equity</td>
<td>3,594,000</td>
<td>0.6425</td>
<td>0.2498</td>
<td>N/A</td>
<td>0.2498</td>
<td>0.1605</td>
</tr>
<tr>
<td>Total</td>
<td>5,594,000</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
<td>0.1777</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculation of Capitalization rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate for net cash flow</td>
</tr>
<tr>
<td>Less sustainable average growth rate</td>
</tr>
<tr>
<td>Capitalization rate for net cash flow for future year</td>
</tr>
<tr>
<td>One plus growth rate</td>
</tr>
<tr>
<td>Capitalization rate for net cash flow for current year</td>
</tr>
</tbody>
</table>

4 Discounted Economic Income Method (Invested Capital Model)

<table>
<thead>
<tr>
<th>Terminal Period</th>
<th>12/31/01</th>
<th>12/31/02</th>
<th>12/31/03</th>
<th>12/31/04</th>
<th>12/31/05</th>
<th>Net cash flow to invested capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capitalization rate for net cash flow</td>
<td>$500,000</td>
<td>$550,000</td>
<td>$605,000</td>
<td>$665,500</td>
<td>$732,050</td>
<td>$783,294</td>
</tr>
<tr>
<td>Terminal value</td>
<td>0.50</td>
<td>1.50</td>
<td>2.50</td>
<td>3.50</td>
<td>4.50</td>
<td>$7,272,925</td>
</tr>
<tr>
<td>Present value of cash flows</td>
<td>460,737</td>
<td>430,339</td>
<td>401,947</td>
<td>375,428</td>
<td>350,659</td>
<td>3,483,799</td>
</tr>
<tr>
<td>Less interest bearing debt</td>
<td>2,000,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of equity</td>
<td>3,502,909</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The iterative process is repeated until the value of the calculated equity in Step 4 equals the value of the equity in Steps 2 and 3.